XV. Mathematics, Grade 10

Grade 10 Mathematics Test

The spring 2017 grade 10 Mathematics test was based on standards in the 2011 *Massachusetts Curriculum Framework for Mathematics* that match content in the grade 9–10 standards from the 2000 *Massachusetts Mathematics Curriculum Framework*. The standards in the 2011 *Framework* on the grade 10 test are organized under the five major conceptual categories listed below.

- Number and Quantity
- Algebra
- Functions
- Geometry
- Statistics and Probability

The *Massachusetts Curriculum Framework for Mathematics* is available on the Department website at <u>www.doe.mass.edu/frameworks/current.html</u>. More information and a list of standards assessable on the spring 2017 test are available at <u>www.doe.mass.edu/mcas/transition/?section=math10</u>.

Mathematics test results for grade 10 are reported under four MCAS reporting categories, which are based on the five *Framework* conceptual categories listed above.

The table at the conclusion of this chapter indicates each item's reporting category, the 2011 *Framework* standard it assesses, and the 2000 *Framework* standard it assesses. The correct answers for multiple-choice and short-answer items are also displayed in the table.

Test Sessions

The grade 10 Mathematics test included two separate test sessions, which were administered on consecutive days. Each session included multiple-choice and open-response items. Session 1 also included short-answer items.

Reference Materials and Tools

Each student taking the grade 10 Mathematics test was provided with a grade 10 Mathematics Reference Sheet. A copy of the reference sheet follows the final question in this chapter.

During Session 2, each student had sole access to a calculator with at least four functions and a square root key. Calculator use was not allowed during Session 1.

During both Mathematics test sessions, the use of bilingual word-to-word dictionaries was allowed for current and former English language learner students only. No other reference tools or materials were allowed.

Grade 10 Mathematics SESSION 1

You may use your reference sheet during this session. You may **not** use a calculator during this session.



DIRECTIONS

This session contains fourteen multiple-choice questions, four short-answer questions, and three open-response questions. Mark your answers to these questions in the spaces provided in your Student Answer Booklet.



The first four terms in a linear sequence are shown below.

1, 7, 13, 19,...

What is the sixth term in the sequence?

- A. 30
- B. 31
- C. 32
- D. 33



The length, in centimeters, of a diagonal of a rectangle is represented by the expression below.

 $\sqrt{11^2 + 14^2}$

Which of the following is closest to the length of the diagonal?

- A. 5 centimeters
- B. 7 centimeters
- C. 18 centimeters
- D. 25 centimeters



The line plot below shows the number of goals scored by a soccer team in each of 12 games.



Number of Goals Scored

Based on the line plot, what is the median number of goals scored by the team in the 12 games?

- A. 1
- B. 2
- C. 3
- D. 4

4 Line g has a slope of $-\frac{4}{7}$. Which of the following equations represents a line that is **perpendicular** to line g?

A.
$$y = -\frac{7}{4}x$$

B. $y = -\frac{4}{7}x$
C. $y = \frac{4}{7}x$
D. $y = \frac{7}{4}x$

5 An art museum has two types of pieces of art: paintings and sculptures.

- The museum has 2,009 paintings.
- The museum has 492 sculptures.

Which of the following is closest to the fraction of the museum's pieces of art that are paintings?

A.
$$\frac{20}{25}$$

B.
$$\frac{20}{24}$$

C. $\frac{21}{24}$

D. $\frac{21}{25}$

A right circular cylinder has a diameter (6)of 10 inches and a height of 3 inches. What is the volume, in cubic inches, of the cylinder?

- Α. 15π
- B. 75π
- C. 225π
- D. 300π

7

Which of the following is equivalent to the expression below?

$$(4x+6)(2x)$$

- A. 16*x*
- B. 20*x*
- C. $8x^2 + 6x$
- D. $8x^2 + 12x$



9

Which of the following is equivalent to the expression below?

$$x^2 - 5x - 24$$

A.
$$(x - 6)(x + 4)$$

- B. (x + 6)(x 4)
- C. (x + 8)(x 3)
- D. (x 8)(x + 3)

The approximate areas of four oceans are shown in the table below.

Ocean Areas

| Ocean | Area (square kilometers) | | |
|----------|-----------------------------|--|--|
| Atlantic | 76,762,000 | | |
| Pacific | 155,557,000 | | |
| Indian | 68,556,000 | | |
| Arctic | 14,056,000 | | |

The area of Lake Superior is approximately 82,100 square kilometers. Based on the table, which ocean has an area that is closest to 1,000 times the area of Lake Superior?

- A. Atlantic
- B. Pacific
- C. Indian
- D. Arctic



The histogram below shows the lengths, in minutes, of the movies shown at Noble Cinema last month.

Movie Lengths at Noble Cinema



Length of Movie (in minutes)

Based on the histogram, which of the following is true?

- A. The median movie length was between 105 and 109 minutes.
- B. The majority of the movies were between 130 and 134 minutes long.
- C. There were a total of twelve movies that were less than 100 minutes long.
- D. The number of movies shorter than 105 minutes was the same as the number of movies longer than 104 minutes.



What is the value of the expression below?

 $\left(\sqrt{11}\right)^4$

- A. 11
- B. 22
- C. 121
- D. 1331
- **12** What are the solutions of the equation below?

$$x^2 + 4x - 12 = 0$$

- A. -6 and -2
- B. -6 and 2
- C. -8 and -4
- D. -8 and 4

- **13** The volume of a cube is 24 cubic inches. Which of the following estimates is closest to the length of each edge of the cube?
 - A. 4.9 inches
 - B. 3.1 inches
 - C. 2.9 inches
 - D. 2.5 inches



Which of the following best represents the graph of the equation below?

 $y = \frac{1}{2}x - 4$







Questions 15 and 16 are short-answer questions. Write your answers to these questions in the boxes provided in your Student Answer Booklet. Do not write your answers in this test booklet. You may do your figuring in the test booklet.

15 What is the value of the expression below?

 $3^2 - 16 \div 4 + 2$



The perimeter of a square is 48 inches. What is the area, in square inches, of the square?

Question 17 is an open-response question.

- BE SURE TO ANSWER AND LABEL ALL PARTS OF THE QUESTION.
- Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.
- If you do the work in your head, explain in writing how you did the work.

Write your answer to question 17 in the space provided in your Student Answer Booklet.

17 Triangle *KLM* is shown on the coordinate grid below.



Copy the axes, the labels, and triangle *KLM* exactly as shown onto the grid in your Student Answer Booklet.

- a. On the grid you copied into your Student Answer Booklet, draw triangle K'L'M', the image of triangle *KLM* after it has been translated 8 units right and 3 units down. Be sure to label the vertices.
- b. On your grid, draw triangle K''L''M'', the image of triangle K'L'M' after it has been reflected over the *x*-axis. Be sure to label the vertices.

Point P(x, y) lies on triangle *KLM*. Point P'' is the image of point P after the transformations from part (a) and part (b) have been completed.

c. Write an expression that represents the *y*-coordinate of point P'' in terms of *y*. Show or explain how you got your answer.

Questions 18 and 19 are short-answer questions. Write your answers to these questions in the boxes provided in your Student Answer Booklet. Do not write your answers in this test booklet. You may do your figuring in the test booklet.

18 The first four terms in a geometric sequence are shown below.

 $\frac{1}{18}, \frac{1}{3}, 2, 12, \ldots$

What is the next term in the sequence?

Write your answer to question 19 in the box provided in your Student Answer Booklet.

19 The line graph below shows the total number of miles traveled by a car during a 7-year period.



Miles Traveled by Car

Based on the line graph, between which two consecutive years was the rate of change, in miles traveled per year, the greatest?

Questions 20 and 21 are open-response questions.

- BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.
- Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.
- If you do the work in your head, explain in writing how you did the work.

Write your answer to question 20 in the space provided in your Student Answer Booklet.



Stuart wrote the expression shown below.

 $16 + 8^2 \div 4 - 4$

- a. What is the value of Stuart's expression? Show or explain how you got your answer.
- b. In your Student Answer Booklet, insert **one set** of parentheses into Stuart's expression so that the value of the expression is undefined. Show or explain how you got your answer.

Talia wrote the expression shown below.

$$(16 + 8^2) \div 4 \cdot 2 - 4$$

Talia found the value of her expression using the following steps:

Step 1: $(16 + 64) \div 4 \cdot 2 - 4$ Step 2: $80 \div 4 \cdot 2 - 4$ Step 3: $80 \div 8 - 4$ Step 4: 10 - 4Step 5: 6

c. Is the value that Talia found for her expression correct? Explain your reasoning.

Talia removed the set of parentheses from her expression to create the new expression shown below.

$$16 + 8^2 \div 4 \cdot 2 - 4$$

d. What is the value of Talia's new expression? Show or explain how you got your answer.

Write your answer to question 21 in the space provided in your Student Answer Booklet.



The table below shows the number of full-time employees at eight guitar-production companies and the number of guitars produced by each company last year.

Guitar Production

| Number of Full- Time Employees | 3 | 5 | 6 | 8 | 8 | 10 | 13 | 18 |
|-----------------------------------|----|-----|-----|-----|-----|-----|-----|-----|
| Number of Guitars Produced | 98 | 189 | 235 | 309 | 336 | 412 | 494 | 692 |

On the grid in your Student Answer Booklet, copy the title, the axes, and the labels exactly as shown below.



Guitar Production

- a. On your grid, create a scatterplot using the data from the table.
- b. On your scatterplot, draw a line of best fit for the data.
- c. Write an equation that represents the line of best fit you drew in part (b). Show or explain how you got your answer.
- d. Use your equation to estimate the number of full-time employees needed if a company plans to produce 1000 guitars in a year. Show or explain how you got your answer.

Grade 10 Mathematics SESSION 2

You may use your reference sheet during this session. You may use a calculator during this session.



DIRECTIONS

This session contains eighteen multiple-choice questions and three open-response questions. Mark your answers to these questions in the spaces provided in your Student Answer Booklet.

22

Shirley is saving money to buy a computer.

- The computer she will buy costs \$1,200.
- She has already saved \$300.

Shirley will save another \$60 each week until she has saved enough money to buy the computer.

How many weeks will it take Shirley to save enough money to buy the computer?

- A. 5
- **B**. 10
- C. 15
- D. 20



The diagram below shows circle O with radii \overline{OL} and \overline{OK} .



The measure of $\angle OLK$ is 35°. What is the measure of $\angle LOK$?

- A. 70°
- B. 90°
- C. 110°
- D. 130°



Muriel and Ramon bought school supplies at the same store.

- Muriel bought 2 boxes of pencils and 4 erasers for a total of \$11.
- Ramon bought 1 box of pencils and 3 erasers for a total of \$7.

Which of the following systems of equations can be used to find p, the price in dollars of one box of pencils, and e, the price in dollars of one eraser?

- A. 2p + 4e = 11p + 3e = 7
- B. 2p + 4e = 7p + 3e = 11
- C. 4p + 2e = 113p + e = 7
- D. 4p + 2e = 73p + e = 11

25

The number of chaperones needed for a school field trip is directly proportional to the number of students going on the field trip. If 96 students are going on the field trip, 12 chaperones are needed.

How many chaperones are needed if 120 students are going on the field trip?

- A. 15
- B. 20
- C. 24
- D. 36

26

The box-and-whisker plot below shows the distribution of student scores on a history test.



Based on the box-and-whisker plot, what is the median history test score?

- A. 72
- B. 76
- C. 80
- D. 88



27 A store sells different-colored pens. The circle graph below represents all pens for sale at the store.



There are 200 black pens for sale at the store. How many blue pens are for sale at the store?

- A. 70
- B. 115
- C. 175
- D. 190



The diagram below shows a 20-foot ladder leaning against a wall. The bottom of the ladder is 10 feet from the base of the wall.



Based on the dimensions in the diagram, what is the value of x?

- A. 15
- B. 30
- C. 45
- D. 60



On a coordinate grid, point *H* is the midpoint of \overline{TW} .

- Point *H* has coordinates (4, -4).
- Point *W* has coordinates (12, 2).

What are the coordinates of point T?

- A. (16, -2)
- B. (8, −1)
- C. (-4, -10)
- D. (-8, -6)

30

The height of a flagpole, rounded to the nearest foot, is 42 feet. Which interval shows the possible values of h, the actual height in feet of the flagpole?

A. $41 \le h < 43$ B. $41 \le h \le 43$ C. $41.5 \le h < 42.5$ D. $41.5 \le h \le 42.5$ 31 The length of a rectangular patio is three times its width. The area of the patio is 432 square feet.

What is the length, in feet, of the patio?

- A. 12B. 18
- C. 24
- D. 36



Which of the following pairs of numbers are additive inverses of each other?

- A. $-0.5 \mbox{ and } -0.5$
- B. -0.5 and 2
- C. 0.5 and -0.5
- D. 0.5 and 2



The scatterplot below shows the relationship between the values of 12 homes and the monthly rent charged for each home.



Based on the line of best fit for the scatterplot, what is the approximate value of a home that has a monthly rent of \$900?

- A. \$125,000
- B. \$200,000
- C. \$275,000
- D. \$450,000



A café uses three different types of bread and three different fillings to make sandwiches. The table below shows the number of sandwiches the café made yesterday.

| | | Sandwich Filling | | | |
|---------------|-----------|------------------|--------|--------|--|
| | | Turkey | Cheese | Veggie | |
| of d | Rye | 40 | 15 | 15 | |
| Type Bread | Wheat | 24 | 20 | 6 | |
| | Sourdough | 16 | 5 | 9 | |

What percent of the **turkey** sandwiches made yesterday were made with wheat bread?

- A. 16%
- B. 24%
- C. 30%
- D. 48%

35 Which of the following scatterplots shows a strong negative correlation between x and y?



Question 36 is an open-response question.

- BE SURE TO ANSWER AND LABEL ALL PARTS OF THE QUESTION.
- Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.
- If you do the work in your head, explain in writing how you did the work.

Write your answer to question 36 in the space provided in your Student Answer Booklet.



Livy and Zack are writing arithmetic sequences. The first three terms of Livy's sequence are shown below.

4, 7, 10, . . .

- a. What is the common difference for Livy's sequence? Show or explain how you got your answer.
- b. Write an expression that can be used to find the *n*th term of Livy's sequence.

The *n*th term of Zack's sequence is three times the *n*th term of Livy's sequence.

- c. What is the 5th term of Zack's sequence? Show or explain how you got your answer.
- d. Write an expression that represents the difference of the nth term of Zack's sequence and the nth term of Livy's sequence.

Mark your answers to multiple-choice questions 37 through 40 in the spaces provided in your Student Answer Booklet. Do not write your answers in this test booklet. You may do your figuring in the test booklet.

Questions 41 and 42 are open-response questions.

- BE SURE TO ANSWER AND LABEL ALL PARTS OF EACH QUESTION.
- Show all your work (diagrams, tables, or computations) in your Student Answer Booklet.
- If you do the work in your head, explain in writing how you did the work.

Write your answer to question 41 in the space provided in your Student Answer Booklet.



Three objects are shown below: a sphere, a right circular cylinder, and a right circular cone.



The radius, r, of each of the objects is 10 inches.

a. What is the volume, in cubic inches, of the sphere? Show or explain how you got your answer.

The height, h, of the cylinder is 2 times its radius.

b. What is the volume, in cubic inches, of the cylinder? Show or explain how you got your answer.

The volume of the cone is equal to the volume of the cylinder.

c. What is the value of k, the height in inches of the cone? Show or explain how you got your answer.

The radius of the sphere will be changed so that the volume of the sphere will be equal to the volume of the cylinder.

d. By how many inches would the radius of the sphere have to change for its volume to be equal to the volume of the cylinder? Show or explain how you got your answer.

Write your answer to question 42 in the space provided in your Student Answer Booklet.



Allen wants to add to his existing collection of 300 bottles. Starting today, he will collect 25 bottles each week.

- a. What will be the **total** number of bottles in Allen's collection after 4 weeks? Show or explain how you got your answer.
- b. Write an expression to represent the total number of bottles in Allen's collection after w weeks.
- c. After how many weeks will Allen have a total of exactly 1,000 bottles in his collection? Show or explain how you got your answer.

Allen's goal is to have between 1,500 and 1,600 bottles in his collection.

d. Write and solve a compound inequality to determine the number of weeks it will take Allen to reach his goal. Show your work.



AREA FORMULAS

| square $A = s^2$ |
|---|
| rectangle $A = bh$ |
| parallelogram $A = bh$ |
| triangle $A = \frac{1}{2}bh$ |
| trapezoid $A = \frac{1}{2}h(b_1 + b_2)$ |
| circle $A = \pi r^2$ |

LATERAL SURFACE AREA FORMULAS

right rectangular prism LA = 2(hw) + 2(lh)right circular cylinder $LA = 2\pi rh$ right circular cone $LA = \pi r\ell$ $(\ell = \text{slant height})$ right square pyramid $LA = 2s\ell$ $(\ell = \text{slant height})$

VOLUME FORMULAS

| cube | $V = s^3$ (s = length of an edge) |
|------------------------|--------------------------------------|
| right rectangular pris | m $V = lwh$ |
| | OR |
| | V = Bh (B = area of a base) |
| sphere | $\dots V = \frac{4}{3}\pi r^3$ |
| right circular cylinde | \mathbf{r} $V = \pi r^2 h$ |
| right circular cone | $\dots V = \frac{1}{3}\pi r^2 h$ |
| right square pyramid | $\dots V = \frac{1}{3}s^2h$ |

CIRCLE FORMULAS

 $C = 2\pi r$ $A = \pi r^2$

TOTAL SURFACE AREA FORMULAS

cube $SA = 6s^2$ right rectangular prismSA = 2(lw) + 2(hw) + 2(lh)sphere $SA = 2\pi r^2$ right circular cylinder $SA = 2\pi r^2 + 2\pi rh$ right circular cone $SA = \pi r^2 + \pi r\ell$ $(\ell = \text{slant height})$ right square pyramid $SA = s^2 + 2s\ell$ $(\ell = \text{slant height})$

SPECIAL RIGHT TRIANGLES



Grade 10 Mathematics Spring 2017 Released Items: Reporting Categories, Standards, and Correct Answers

| Item No. | Page No. | Reporting Category ¹ | Standard ¹ | Correct Answer ² (MC/SA) | 2000 Standard ³ |
|----------|----------|---------------------------------|-----------------------|--|-------------------------------|
| 1 | 201 | Algebra & Functions | F.BF.1.02 | В | 10.P.1 |
| 2 | 201 | Number and Quantity | 8.NS.1.02 | С | 10.N.3 |
| 3 | 201 | Statistics & Probability | S.ID.1.01 | В | 10.D.1 |
| 4 | 202 | Geometry | G.GPE.2.05 | D | 10.G.8 |
| 5 | 202 | Number and Quantity | 7.EE.2.03 | A | 10.N.4 |
| 6 | 202 | Geometry | G.GMD.1.03 | В | 10.M.2 |
| 7 | 202 | Algebra & Functions | A.APR.1.01 | D | 10.P.3 |
| 8 | 203 | Algebra & Functions | A.SSE.1.02 | D | 10.P.4 |
| 9 | 203 | Number and Quantity | 7.EE.2.03 | A | 10.N.4 |
| 10 | 203 | Statistics & Probability | S.ID.1.01 | С | 10.D.1 |
| 11 | 204 | Number and Quantity | N.RN.1.02 | С | 10.N.1 |
| 12 | 204 | Algebra & Functions | A.REI.2.04 | В | 10.P.5 |
| 13 | 204 | Number and Quantity | 8.NS.1.02 | C | 10.N.3 |
| 14 | 205 | Algebra & Functions | F.IF.3.08 | A | 10.P.2 |
| 15 | 206 | Number and Quantity | 7.EE.2.03 | 7 | 10.N.2 |
| 16 | 206 | Geometry | 7.G.2.06 | 144 square inches | 10.M.1 |
| 17 | 207 | Geometry | G.CO.1.05 | | 10.G.9 |
| 18 | 208 | Algebra & Functions | F.BF.1.02 | 72 | 10.P.1 |
| 19 | 209 | Statistics & Probability | S.ID.3.07 | years 2 and 3 | 10.D.1 |
| 20 | 210 | Number and Quantity | 7.EE.2.03 | | 10.N.2 |
| 21 | 211 | Statistics & Probability | S.ID.2.06 | | 10.D.2 |
| 22 | 212 | Algebra & Functions | A.CED.1.01 | С | 10.P.7 |
| 23 | 212 | Geometry | G.C.1.02 | C | 10.G.3 |
| 24 | 213 | Algebra & Functions | A.CED.1.02 | A | 10.P.8 |
| 25 | 213 | Algebra & Functions | A.CED.1.01 | A | 10.P.7 |
| 26 | 213 | Statistics & Probability | S.ID.1.01 | В | 10.D.1 |
| 27 | 214 | Statistics & Probability | 6.SP.2.04 | C | 10.D.1 |
| 28 | 214 | Geometry | G.SRT.3.06 | В | 10.G.6 |
| 29 | 215 | Geometry | G.GPE.2.06 | C | 10.G.7 |
| 30 | 215 | Number and Quantity | N.Q.1.03 | C | 10.M.4 |
| 31 | 215 | Algebra & Functions | A.CED.1.01 | D | 10.P.7 |
| 32 | 215 | Number and Quantity | 7.NS.1.03 | C | 10.N.1 |
| 33 | 216 | Statistics & Probability | S.ID.2.06 | В | 10.D.2 |
| 34 | 216 | Statistics & Probability | S.ID.2.05 | C | 10.D.1 |
| 35 | 217 | Statistics & Probability | S.ID.2.06 | D | 10.D.1 |
| 36 | 218 | Algebra & Functions | F.BF.1.02 | | 10.P.1 |
| 37 | 219 | Geometry | 7.G.2.04 | D | 10.M.3 |
| 38 | 219 | Number and Quantity | 7.EE.2.03 | A | 10.N.1 |
| 39 | 219 | Geometry | 8.G.1.05 | В | 10.G.5 |
| 40 | 219 | Geometry | 7.G.2.06 | В | 10.M.1 |
| 41 | 220 | Geometry | G.GMD.1.03 | | 10.M.2 |
| 42 | 221 | Algebra & Functions | A.REI.2.03 | | 10.P6 |

¹ The Reporting Category and Standard columns refer to the current (2011) *Massachusetts Curriculum Framework for Mathematics*. More information about reporting categories for Mathematics is available on the Department's website at <u>www.doe.mass.edu/mcas/tdd/math.html?section=testdesign</u>.

² Answers are provided here for multiple-choice and short-answer items only. Sample responses and scoring guidelines for open-response items, which are indicated by the shaded cells, will be posted to the Department's website later this year.

³ The Department is providing the standard from the previous (2000) curriculum framework for Mathematics for reference purposes.